SEMESTER-1 (Period-I)

P11CH1

TOPIC

1 Motion in Two Dimensions

1.1. SCALAR AND VECTOR QUANTITIES

All those quantities which can be measured are known as ***physical quantities****.* These quantities can be broadly classified into two categories—scalar quantities and vector quantities.

Scalar quantities *are those physical quantities which have only magnitude and no direction.*

These obey the ordinary laws of Algebra. A scalar quantity is completely specified by merely stating a number. A few *examples* of scalars are volume, mass, speed, density, temperature, pressure, time, power, total path length and energy.

Vector quantities *are those physical quantities which have both magnitude and direction and obey the laws of vector addition.*

A vector is specified not by merely stating a number but a direction as well. Since the concept of vectors involves the idea of direction, therefore, vectors do not follow the ordinary laws of Algebra. A few examples of vectors are displacement, velocity, acceleration, impulse, force and linear momentum.

1.2. REPRESENTATION OF A VECTOR

A vector is represented by a line with an arrow head. In Fig. 1.1, a vector $\vec{a}$ is represented by a directed line PQ. The length of the line gives the magnitude of the vector. The magnitude of the vector is called the modulus of the vector. The direction of the arrow represents the direction of the vector.

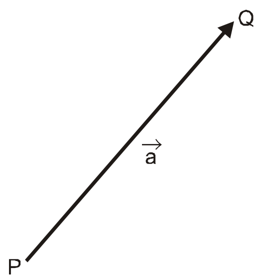


Fig. 1.1. Representation of a vector

1.3. IMPORTANT TERMS

(*i*) ***Parallel vectors.*** If two collinear vectors $\vec{a}$ and $\vec{b}$ act in the same direction, then the angle between them is 0°. When vectors act along the same direction, they are called parallel vectors.

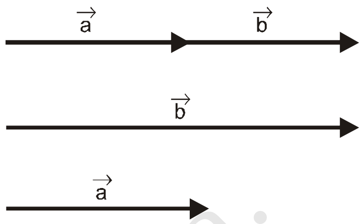


Fig. 1.2. Parallel vectors

(*ii*) ***Antiparallel vectors.*** If two collinear vectors act in opposite directions, then the angle between them is 180° or π radian. Vectors are said to be anti-parallel if they act in opposite directions.

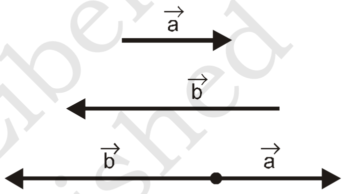


Fig. 1.3. Antiparallel vectors

(*iii*) ***Unit vector*** *of a given vector is a vector of unit magnitude and has the same direction as that of the given vector.*

Unit vector is used to denote the direction of a given vector. It is unitless and dimensionless vector.

Any vector $\vec{a}$ can be expressed in terms of its unit vector $\hat{a}$ as follows:

$$\vec{a} = a\,\hat{a}$$

Here $\hat{a}$ is in the direction of $\vec{a}$. $\hat{a}$ is read as 'a hat' or 'a cap'.

$$\hat{a} = \frac{\vec{a}}{a} \quad \text{or} \quad \frac{\vec{a}}{|\vec{a}|}$$

So, if a given vector is divided by its magnitude, we get a unit vector.

(*iv*) The ***three rectangular unit vectors*** $\hat{i}$, $\hat{j}$ and $\hat{k}$ are shown in Fig. 1.4. $\hat{i}$ denotes the direction of X-axis. $\hat{j}$ denotes the direction of Y-axis and $\hat{k}$ denotes the direction of Z-axis. The three unit vectors $\hat{i}$,

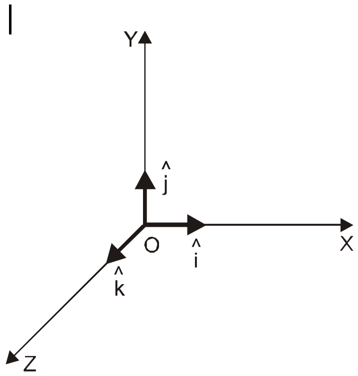


Fig. 1.4. Orthogonal triad of unit vectors

$\hat{j}$ and $\hat{k}$ are collectively known as '*orthogonal triad of unit vectors*'. These are also known as *base vectors.*

(*v*) ***Negative of a vector.*** *A vector is said to be* **negative** *of a given vector if its magnitude is the same as that of the given vector but direction is reversed.*

The negative of a vector $\vec{a}$ is denoted by '$-\vec{a}$'.

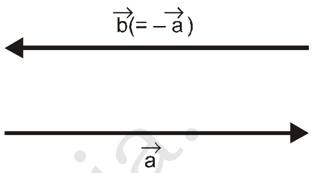


Fig. 1.5. Negative vector

In Fig. 1.5, $\vec{b}$ is the negative of $\vec{a}$. $\vec{b} = -\vec{a}$

1.4. POSITION VECTOR

A vector which gives the position of a point with reference to the origin of the co-ordinate system is called ***position vector.***

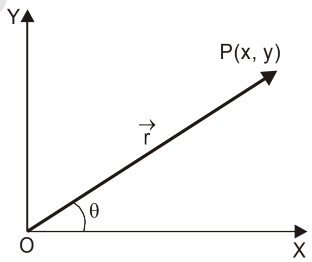


Fig. 1.6. Position vector in two dimensions

Consider a particle moving in a plane. To describe the position of this particle at any time *t*, we use a vector called *position vector.* This *helps to locate the position* of a particle moving in plane or even in space. Suppose at any instant of time, the particle is at P. Then $\vec{OP}$ is the position vector which gives the position of the particle with reference to a point O in the plane of motion. This point O has been chosen as the origin.

The magnitude of the position vector gives the distance of the particle from some arbitrarily chosen origin. In addition to this, the direction of the position vector gives us the direction θ in which P lies as viewed from O.

It may be noted here that position vectors will be different for different positions of the particle.

The position vector $\vec{r}$ at any time *t*, in terms of co-ordinates *x* and *y*, is given by,

$$\vec{r} = \vec{x} + \vec{y} \quad \text{or} \quad \vec{r} = x\,\hat{i} + y\,\hat{j}$$

In magnitude, $|\vec{r}|$ or $r = \sqrt{x^2 + y^2}$

If the position of a point P is chosen with reference to the origin of the three-dimensional rectangular co-ordinate system as shown in Fig. 1.7, then the position vector is given by,

$$\vec{r} = \vec{x} + \vec{y} + \vec{z}$$

or $$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

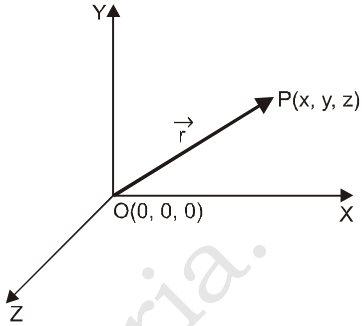


Fig. 1.7. Position vector in three dimensions

The magnitude or modulus of $\vec{r}$ *is given by r or* $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

1.5. EQUALITY OF VECTORS

Two vectors are said to be ***equal*** *if they have the same magnitude and same direction.* In Fig. 1.8, three equal vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ have been represented. The equality of vectors is represented as follows:

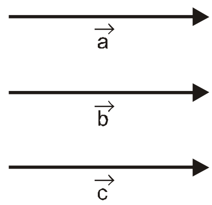


Fig. 1.8. Equal vectors

$$\vec{a} = \vec{b} = \vec{c}$$

Since the three vectors are pointing in the same direction,

$$\therefore \qquad \hat{a} = \hat{b} = \hat{c}$$

Also, since the three vectors have equal magnitudes,

$$\therefore \qquad |\vec{a}| = |\vec{b}| = |\vec{c}|$$

If the scales selected for the representation of three vectors are the same, then three equal vectors are represented by three arrows of equal lengths, pointing in the same direction.

1.6. THE ZERO VECTOR AND ITS PROPERTIES

Zero vector *or* ***null vector*** *is a vector which has zero magnitude and an arbitrary direction.* It is represented by $\vec{0}$.

If $\mu = -\lambda$, then the vector $(\lambda + \mu)\vec{a}$ is equal to $\vec{0}$.

If we multiply a vector by zero, what do we get? The answer is obviously zero vector. Now, let us consider a vector $(\vec{a} + \vec{b})$. If $\vec{b} = -\vec{a}$, then $\vec{a} + \vec{b} = \vec{0}$.

(***i***) The displacement of a ball thrown up and received back by the thrower is a zero vector.

(***ii***) The velocity vector of a stationary body is a zero vector.

1.7. ADDITION OR COMPOSITION OF VECTORS

The process of adding two or more than two vectors is called ***'addition or composition of vectors'.***

When two or more than two vectors are added, we get a single vector called resultant vector.

The ***resultant*** *of two or more than two vectors is a single vector which produces the same effect as the individual vectors together produce.*

Following three laws have been evolved for the addition of vectors.

(***i***) Triangle law of vectors (*for addition of two vectors*)

(***ii***) Parallelogram law of vectors (*for addition of two vectors*)

(***iii***) Polygon law of vectors (*for addition of more than two vectors*).

Triangle Law of Vectors

Let a particle be at the points A, B and C at three successive times t, t' and t'' respectively. $\vec{AB}$ is the displacement vector from time t to t'. $\vec{BC}$ is the displacement vector from time t' to time t''. The total displacement vector $\vec{AC}$ is the sum or the resultant of individual displacement vectors $\vec{AB}$ and $\vec{BC}$.

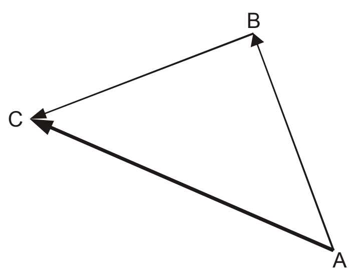


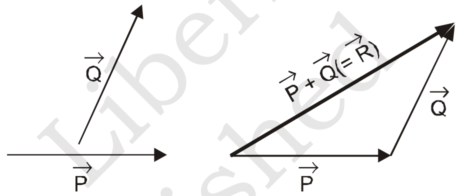
Fig. 1.9. Triangle law of vectors

$$\therefore \quad \vec{AC} = \vec{AB} + \vec{BC}$$

This leads us to the following law known as *triangle law of vectors.* This law is used for the addition of two vectors.

If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.

Suppose we have to add two vectors $\vec{P}$ and $\vec{Q}$ as shown in Fig. 1.10 (*a*). Now, displace $\vec{Q}$ parallel to itself in such a way that the tail of $\vec{Q}$ touches the tip of $\vec{P}$. Complete the triangle to get a new vector $(\vec{P} + \vec{Q})$ running straight from the tail of $\vec{P}$ to the tip of $\vec{Q}$. According to triangle law of vectors, this new vector is the resultant $\vec{R}$ of the given vectors $\vec{P}$ and $\vec{Q}$ such that,



(a) (b)

Fig. 1.10. Triangle law of vectors (Graphical method for addition of vectors)

$$\vec{R} = \vec{P} + \vec{Q}.$$

Triangle law of vectors is applicable to triangle of any shape.

It follows from triangle law of vectors that if three vectors are represented by the three sides of a triangle taken in order, then their resultant is zero. Thus, if three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be represented completely by the three sides of a triangle taken in order, then their vector sum is zero.

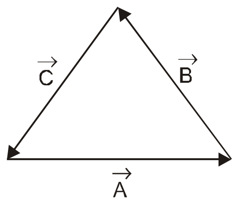


Fig. 1.11

$$\therefore \qquad \vec{A} + \vec{B} + \vec{C} = \vec{0}$$

Parallelogram Law of Vectors

Consider two vectors $\vec{P}$ and $\vec{Q}$ as shown in Fig. 1.12 (*a*). Displace $\vec{Q}$ parallel to itself till the tail of $\vec{Q}$ touches the tail of $\vec{P}$.

Complete the parallelogram as shown in Fig. 1.12 (*b*). Applying triangle law of vectors to the vector triangle OAC, we get

$$\vec{OA} + \vec{AC} = \vec{OC}$$

or $$\vec{P} + \vec{Q} = \vec{R}$$

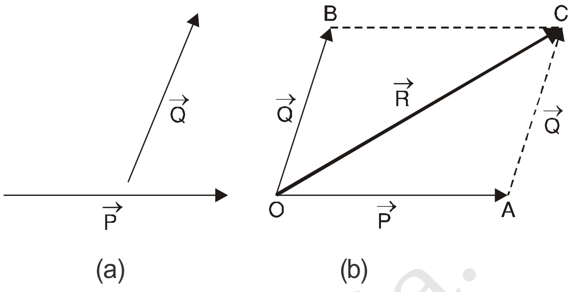


Fig. 1.12. Parallelogram law of vectors

So, we conclude that if two vectors are represented completely by the two adjacent sides, of a parallelogram, drawn from a point, then the diagonal of the parallelogram drawn through that point gives the resultant vector. This is parallelogram law of vectors. It is stated as follows:

"If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram passing through that point."

In Fig. 1.13, two vectors $\vec{P}$ and $\vec{Q}$ are completely represented by the two sides OA and OB respectively of a parallelogram. Then, according to parallelogram law of vectors, the diagonal OC of the parallelogram will give the resultant $\vec{R}$ such that $\vec{R} = \vec{P} + \vec{Q}$.

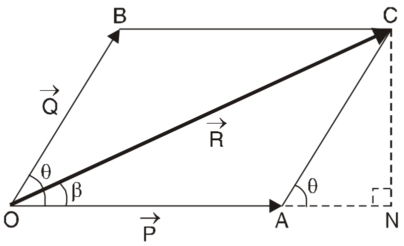


Fig. 1.13. Addition of vectors by parallelogram law of vectors

Let us analytically calculate the magnitude and direction of the resultant vector $\vec{R}$.

Let θ be the angle between two given vectors $\vec{P}$ and $\vec{Q}$. From C, drop a perpendicular CN on OA (produced). In the right-angled ΔANC,

$$\sin\theta = \frac{CN}{AC} \text{ or } CN = AC \sin\theta$$

or $\quad CN = Q \sin \theta \quad [\because \ AC = OB = Q] \quad ...(1)$

Also, $\cos \theta = \dfrac{AN}{AC}$ or $AN = AC \cos \theta = Q \cos \theta$

Now, $ON = OA + AN = P + Q \cos \theta \quad ...(2)$

Considering the right-angled ΔONC,

$$OC^2 = ON^2 + CN^2$$

or
$$R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2$$
[From (2) and (1)]

or
$$R^2 = P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta$$
$$= P^2 + Q^2 \cos^2 \theta + Q^2 \sin^2 \theta + 2PQ \cos \theta$$
$$= P^2 + Q^2 (\cos^2 \theta + \sin^2 \theta) + 2PQ \cos \theta$$

$\therefore$
$$R^2 = P^2 + Q^2 + 2PQ \cos \theta \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

or
$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \quad ...(3)$$

which is the required expression for the magnitude of the resultant of two vectors $\vec{P}$ and $\vec{Q}$ inclined to each other at an angle θ. Equation (3) is known as the **law of cosines.**

Let β be the angle which the resultant $\vec{R}$ makes with $\vec{P}$.

Then,
$$\tan \beta = \frac{CN}{ON} \quad \text{(in } rt. \angle d \ \Delta ONC)$$

or
$$\tan \beta = \frac{Q \sin \theta}{P + Q \cos \theta} \quad \text{[from (2) and (1)]} \quad ...(4)$$

or
$$\beta = \tan^{-1} \left(\frac{Q \sin \theta}{P + Q \cos \theta} \right)$$

which gives the direction of the resultant vector.

SPECIAL CASES

Case I. ***When the given vectors $\vec{P}$ and $\vec{Q}$ act in the same direction***

In this case, $\theta = 0°$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos 0°} \quad \text{[from equation (3)]}$$

$$= \sqrt{P^2 + Q^2 + 2PQ} \quad [\because \cos 0° = 1]$$

$$= \sqrt{(P+Q)^2} = P + Q$$

or $$|\vec{R}| = |\vec{P}| + |\vec{Q}|$$

So, *the magnitude of the resultant vector is equal to the sum of the magnitudes of the given vectors.*

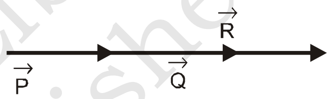


Fig. 1.14

Also, $$\tan\beta = \frac{Q\sin 0°}{P + Q\cos 0°} \quad \text{[from equation (4)]}$$

or $$\tan\beta = 0 \quad [\because \sin 0° = 0]$$

$$\therefore \quad \beta = 0°$$

So, *the resultant vector points in the direction of the given vectors.*

Case II. ***When the given vectors $\vec{P}$ and $\vec{Q}$ act at right angles to each other***

In this case, $\theta = 90°$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$$

or $$R = \sqrt{P^2 + Q^2} \quad [\because \cos 90° = 0]$$

or $$|\vec{R}| = \sqrt{|\vec{P}|^2 + |\vec{Q}|^2}$$

Also, $$\tan\beta = \frac{Q\sin 90°}{P + Q\cos 90°}$$

or $$\tan\beta = \frac{Q}{P} \quad [\because \sin 90° = 1]$$

or $$\beta = \tan^{-1}\left(\frac{Q}{P}\right)$$

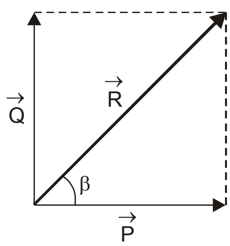


Fig. 1.15

If P = Q, then $R = \sqrt{P^2 + P^2}$ or $R = \sqrt{2P^2} = \sqrt{2}\ P$

Also, in this case, $\tan\beta = \dfrac{P}{P} = 1$ or $\beta = 45°$

Case III. ***When the given vectors $\vec{\mathbf{P}}$ and $\vec{\mathbf{Q}}$ act in opposite directions***

In this case, $\theta = 180°$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°}$$

$$= \sqrt{P^2 + Q^2 - 2PQ} \quad [\because \cos 180° = -1]$$

$$= \sqrt{(P - Q)^2}$$

$$\therefore \quad R = \pm (P - Q) = P - Q \text{ or } Q - P$$

or $$|\vec{R}| = |\vec{P}| \sim |\vec{Q}|$$

[$|\vec{P}| \sim |\vec{Q}|$ implies positive difference between $|\vec{P}|$ and $|\vec{Q}|$.]

So, *the magnitude of the resultant vector is equal to the positive difference of the magnitudes of the given vectors.*

Also, $$\tan\beta = \frac{Q\sin 180°}{P + Q\cos 180°}$$

or $$\tan\beta = 0$$

$$\therefore \quad \beta = 0° \text{ or } 180°$$

When $|\vec{P}| > |\vec{Q}|$, then $\beta = 0°$. [Fig. 1.16]

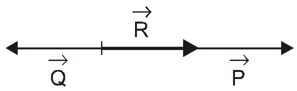


Fig. 1.16

When $|\vec{P}| < |\vec{Q}|$, then $\beta = 180°$. [Fig. 1.17]

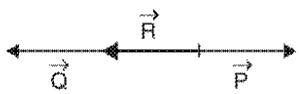


Fig. 1.17

Clearly, *the resultant vector acts in the direction of the bigger of the two vectors.*

Illustrations of Parallelogram Law of Vectors

1. Flight of a bird. When a bird flies, its wings W_1 and W_2 push the air downwards with forces F_1 and F_2 respectively. The air offers equal and opposite reactions R_1 and R_2 in accordance with Newton's third law of motion. According to parallelogram law of vectors, the resultant R of R_1 and R_2 acts on the bird in the upward direction [Fig. 1.18]. This helps the bird to fly upward.

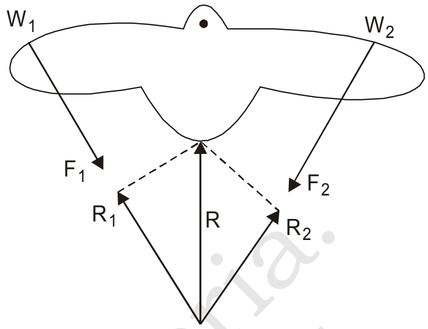


Fig. 1.18

2. Working of a sling. A sling is a Y-shaped metallic or wooden frame to which a rubber band is attached. Tensions T_1 and T_2 are produced in the rubber band when a stone held on the rubber band is pulled [Fig. 1.19]. The resultant of T_1 and T_2 is T in accordance with parallelogram law of vectors. When the stone is released, it moves under the action of T with high speed.

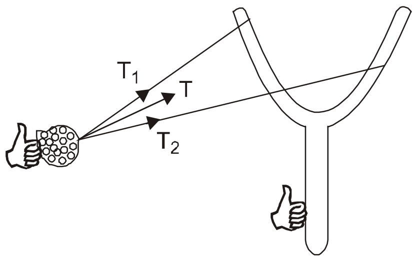


Fig. 1.19

Polygon Law of Vectors

Polygon law of vectors is used for the addition of more than two vectors.

Consider four vectors $\vec{P}$, $\vec{Q}$, $\vec{S}$ and $\vec{T}$ as shown in Fig. 1.20 (*a*). Displace $\vec{Q}$ parallel to itself till the tail of $\vec{Q}$ touches the tip of $\vec{P}$. Similarly, displace $\vec{S}$ parallel to itself till the tail of $\vec{S}$ touches the tip of $\vec{Q}$. Again, displace $\vec{T}$ parallel to itself so that its tail touches the tip of $\vec{S}$. Now a vector $\vec{R}$ running straight from the tail of $\vec{P}$ to the tip of $\vec{T}$ will be the resultant of $\vec{P}$, $\vec{Q}$, $\vec{S}$ and $\vec{T}$.

This is polygon law of vectors stated as follows :

"If a number of vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the sides of an open

convex polygon taken in the same order, then the resultant is represented completely in magnitude and direction by the closing side of the polygon, taken in the opposite order."

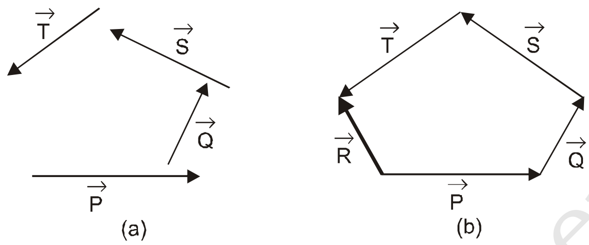


Fig. 1.20. Polygon law of vectors

1.8. PROPERTIES OF VECTOR ADDITION

A quantity can be a vector only if it obeys the laws of vector addition.

Following are the important properties of vector addition.

(*i*) Vectors of the same nature alone can be added. A force vector can be added to force vector only. It cannot be added to displacement vector.

(*ii*) Vector addition is commutative. *The sum of the vectors remains the same in whatever order they may be added.*

According to commutative law of vector addition,

$$\vec{a} + \vec{b} + \vec{c} + \ldots\ldots = \vec{b} + \vec{a} + \vec{c} + \ldots\ldots = \vec{c} + \vec{a} + \vec{b} + \ldots\ldots$$

The result of vector addition does not depend on the order in which the vector sum is written.

(*iii*) Vector addition is distributive.

According to distributive law of vector addition,

$$\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$$

(*iv*) Vector addition is associative. *The sum of the vectors remains the same in whatever grouping they are added.*

According to associative law of vector addition,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

1.9 EQUILIBRANT VECTOR

Equilibrant vector *is a single vector which balances two or more than two vectors acting simultaneously at a point.*

The equilibrant and the resultant vectors are equal in magnitude and opposite in direction.

Example 1. *Resultant of two vectors* $\vec{a}$ *and* $\vec{b}$ *inclined at an angle* θ *is* $\vec{c}$. *Calculate* θ.

Given : $|\vec{a}| = |\vec{b}| = |\vec{c}|$

Solution. $c^2 = a^2 + b^2 + 2ab\cos\theta$

or $c^2 = c^2 + c^2 + 2c^2\cos\theta$ $\quad [\because \ a = b = c]$

or $c^2 = 2c^2(1 + \cos\theta)$

or $1 + \cos\theta = \frac{1}{2}$ $\quad \cos 120° = -\frac{1}{2}$

or $\cos\theta = -\frac{1}{2}$ or $\theta = \mathbf{120°}$

1.10. SUBTRACTION OF VECTORS

Subtraction of a vector $\vec{B}$ *from a vector* $\vec{A}$ *is the addition of vector* $(-\vec{B})$ *to vector* $\vec{A}$.

The subtraction of two vectors often becomes necessary in connection with velocities and accelerations. It is, of course, not very common in the case of forces.

The process of subtracting one algebraic quantity from another is equivalent to adding the negative of the quantity to be subtracted.

$$a - b = a + (-b)$$

In the same manner, the process of subtracting one vector quantity from the other is equivalent to adding vectorially the negative of the vector to be subtracted. Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

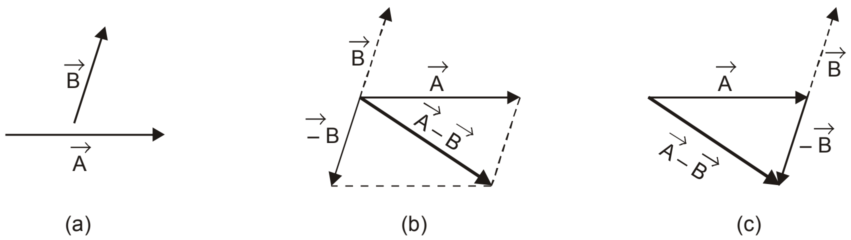


Fig. 1.21. Subtraction of vectors

1.11. PROJECTILE

A body which is in flight through the atmosphere under the influence of gravity alone without being propelled by any fuel is called a ***projectile.***

Examples:

(*i*) A bomb released from an aeroplane in level flight.

(*ii*) A bullet fired from a gun.

(*iii*) A javelin thrown by an athlete.

(*iv*) An arrow released from bow.

(*v*) A stone thrown horizontally from the top of a building.

The path followed by a projectile is called ***trajectory.***

The motion of a projectile is a two-dimensional motion.

1.12 TWO TYPES OF PROJECTILES

Following are the two types of projectiles.

(*i*) Horizontal Projectile. If a body is projected horizontally from a certain height with a certain velocity, then the body is called a horizontal projectile.

(*ii*) Oblique Projectile. If a body is projected at a certain angle with the horizontal, then the body is called an oblique projectile.

1.13. PRINCIPLE OF PHYSICAL INDEPENDENCE OF MOTIONS

The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts.

(*i*) horizontal motion **(*ii*)** vertical motion.

These two motions take place independent of each other. This is called the principle of physical independence of motions.

At any instant, the velocity of a projectile has two components **(*i*)** horizontal component **(*ii*)** vertical component.

The horizontal component remains unchanged throughout the flight. The vertical component is continuously affected by the force of gravity. Thus, while the horizontal motion is a uniform motion, the vertical motion is a uniformly accelerated motion.

1.14. HORIZONTAL PROJECTILE

(*i*) Nature of Trajectory. Consider a projectile thrown horizontally from a point O, with horizontal velocity $\vec{u}$, at a certain height above the ground.

Through the point O, take two axes—X-axis and Y-axis. Let x and y be the horizontal and vertical distances respectively covered by the projectile in time t. At time t, the projectile is at P (Fig. 1.22).

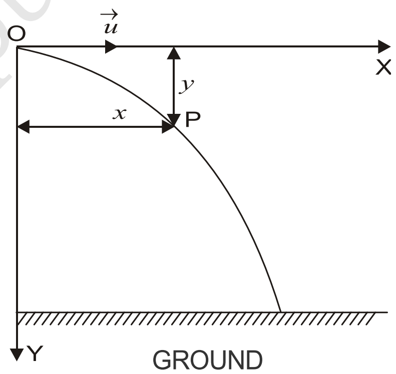


Fig. 1.22. Trajectory of horizontal projectile

The horizontal motion of the projectile is uniform motion. This is because the only force acting on the projectile is force of gravity. This force acts in the vertically downward direction and its horizontal component is zero.

Using $$x = x_0 + u_x t + \frac{1}{2} a_x t^2, \text{ we get}$$

$$x = 0 + ut + 0 \quad \text{or} \quad x = ut \quad \text{or} \quad t = \frac{x}{u} \qquad \ldots(1)$$

The **vertical motion** of the projectile is controlled by force of gravity and is an accelerated motion. The initial velocity u_y in the vertically downward direction is zero. Since Y-axis is taken downwards, therefore, the downward direction will be regarded as positive direction. So, the acceleration a_y of the projectile is + g.

Using $y = y_0 + u_y t + \frac{1}{2} a_y t^2$, we get

$\therefore$ $y = 0 + 0 + \frac{1}{2} g t^2$ or $y = \frac{1}{2} g t^2$ ...(2)

Combining (1) and (2), we get

$$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2 \text{ or } y = \frac{g}{2u^2} x^2 \text{ or } y = kx^2 \quad ...(3)$$

where $k \left(= \frac{g}{2u^2}\right)$ is a constant.

Equation (3) is a second degree equation in x and a first degree equation in y. This is the equation of a parabola.

In the study of projectile motion, both position and time are measured from 'O'.

$\therefore$ $x_0 = y_0 = 0$.

CONCLUSION

A body thrown horizontally from a certain height above the ground follows a parabolic trajectory till it hits the ground.

(*ii*) Time of Flight (T). *It is the time of descent of the projectile from the point of projection to the ground. It is the total time for which the projectile is in flight.*

Let h be the vertical height of the point of projection above the ground.

Considering vertically downward motion,

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

Putting values, $h = 0 + 0 + \frac{1}{2} g T^2$ or $T = \sqrt{\frac{2h}{g}}$

(*iii*) Horizontal Range (R). *It is the horizontal distance travelled by the projectile during the time of flight.*

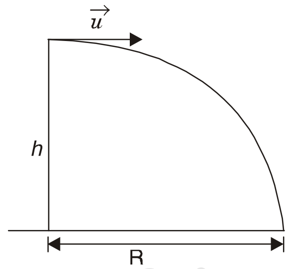


Fig. 1.23

Using $x = x_0 + u_x t + \frac{1}{2} a_x t^2$, we get

$$R = 0 + uT + 0$$

$$= uT = u\sqrt{\frac{2h}{g}}.$$

1.15. TRAJECTORY OF AN OBLIQUE PROJECTILE

Consider a projectile thrown with velocity u at an angle θ with the horizontal (Fig. 1.24). The velocity u can be resolved into two rectangular components (*i*) $u \cos \theta$ along X-axis and (*ii*) $u \sin \theta$ along Y-axis. The motion of the projectile is a two-dimensional motion. It can be supposed to be made up of two motions-horizontal motion (along X-axis) and vertical motion (along Y-axis). The **horizontal motion** of the projectile is uniform motion. This is because the only force acting on the projectile is the force of gravity. This force acts in the vertically downward direction and its horizontal component is zero. Thus, the equations of motion of the projectile for the horizontal direction are simply the equations of uniform motion in a straight line. The horizontal motion takes place with constant velocity $u \cos \theta$. If x be the horizontal distance covered in time t, then

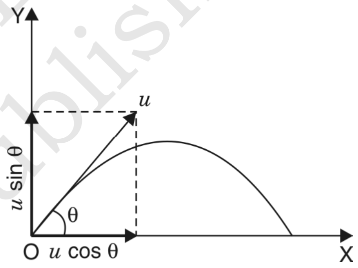


Fig. 1.24. Trajectory of an oblique projectile

$$x = (u \cos \theta)\, t$$

or

$$t = \frac{x}{u \cos \theta} \quad \ldots(1)$$

The **vertical motion** of the projectile is controlled by the force of gravity. The projectile increases its height up to a maximum where its vertical velocity v_y becomes zero. After this, the projectile reverses its vertical direction and returns to earth striking the ground with a speed u which is the same as the initial speed of the projectile.

Let y be the vertical distance covered by the projectile in time t. Let us now consider the vertical motion of the projectile.

$$u_y = u \sin \theta,\ a_y = -g,\ 't' = t$$

We know that $$y = u_y t + \frac{1}{2} a_y t^2$$

Substituting values, $$y = u \sin \theta\ t - \frac{1}{2} g t^2$$

Using equation (1), $$y = u \sin \theta \left(\frac{x}{u \cos \theta}\right) - \frac{1}{2} g \left(\frac{x}{u \cos \theta}\right)^2$$

or $$\boxed{y = x \tan \theta - \frac{g x^2}{2u^2 \cos^2 \theta}} \qquad \ldots(2)$$

This is a first degree equation in y and a second degree equation in x. This is the equation of a *parabola. So, the path followed by the projectile, *i.e.*, **the trajectory of the projectile is parabolic.**

It is clear from equation (2) that the trajectory is completely known if u and θ are known. It should be kept in mind that equation (2) is valid only if θ lies between 0 and $\pi/2$.

1.16. MAXIMUM HEIGHT

It is the maximum height to which a projectile rises above the horizontal plane of projection. It is denoted by $h_{max.}$ or H. It is also known as vertical range.

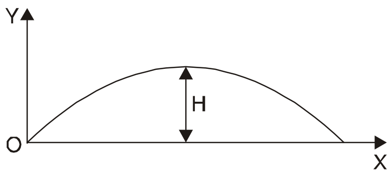


Fig. 1.25. Maximum height of an oblique projectile

In order to calculate the maximum height H, we make use of the fact that the velocity v_y of the projectile at the maximum height is zero. If t_1 be the time taken by the projectile to reach maximum height, then using equation $v_y = u_y + a_y t$, we get

$$0 = u \sin \theta - g t_1 \quad \text{or} \quad g t_1 = u \sin \theta \quad \text{or} \quad t_1 = \frac{u \sin \theta}{g}$$

When 't' = t_1, y = H.

$$y_0 = 0,\ u_y = u \sin \theta,\ a_y = -g$$

*The general equation of a parabola which passes through the origin is $y = ax - bx^2$.

Using relation $y = y_0 + u_y\, t + \frac{1}{2}\, a_y\, t^2$, we get

$$H = (u \sin\theta)\, t_1 - \frac{1}{2} g t_1^2$$

or $$H = u \sin\theta \times \frac{u \sin\theta}{g} - \frac{1}{2}\, g \left(\frac{u \sin\theta}{g}\right)^2$$

or $$H = \frac{u^2 \sin^2\theta}{g} - \frac{u^2 \sin^2\theta}{2g} \quad \text{or} \quad \boxed{H = \frac{u^2 \sin^2\theta}{2g}}$$

1.17. TIME OF FLIGHT

***Time of flight** is the total time taken by the projectile to return to the same level from where it was thrown.* It is the total time for which the projectile is in flight.

Time of flight is equal to twice the time taken by the projectile to reach the maximum height. This is because the time of ascent is equal to the time of descent. This fact is also clear from the symmetry of the curve.

Time of flight, $T = 2t$

where *t is the time taken by the projectile to reach maximum height.

Now, $v_y = 0,\ a_y = -g,\ u_y = u \sin\theta$

We know that $v_y = u_y + a_y t$

Substituting values, $0 = u \sin\theta - gt$ or $gt = u \sin\theta$

or $$t = \frac{u \sin\theta}{g}$$

$\therefore$ $$\boxed{T = \frac{2u \sin\theta}{g}}$$

1.18. HORIZONTAL RANGE

***Horizontal range** is the total horizontal distance from the point of projection to the point where the projectile comes back to the plane of projection.* It is denoted by R.

---

* This is the time of ascent of the projectile.

In order to calculate horizontal range R, we shall consider horizontal motion of the projectile. The horizontal motion is uniform motion. It takes place with constant velocity $u \cos \theta$.

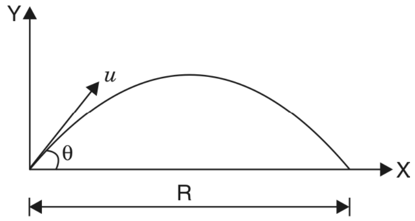


Fig. 1.26. Horizontal range of an oblique projectile

$$\therefore \quad R = u \cos \theta \times \text{time of flight}$$

$$= u \cos \theta \times \frac{2u \sin \theta}{g}$$

or $$R = \frac{u^2 (2 \sin \theta \cos \theta)}{g}$$

or $$\boxed{R = \frac{u^2 \sin 2\theta}{g}} \qquad (\because 2 \sin \theta \cos \theta = \sin 2\theta)$$

1.19. MAXIMUM HORIZONTAL RANGE

For a given velocity of projection and at a given place, the value of R will be maximum when the value of sin 2θ is maximum *i.e.,* 1.

For R to be maximum, $\sin 2\theta = 1$ (maximum value)

or $$\sin 2\theta = \sin 90°$$

or $$\theta = 45°$$

So, for a given velocity, the angle of projection for maximum range is 45°, *i.e.,* $\frac{x}{4}$.

Maximum horizontal range, $\boxed{R_{max.} = \frac{u^2}{g}}$

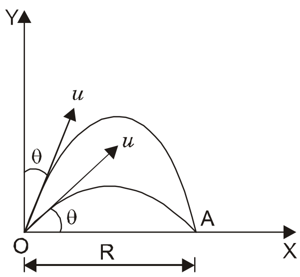


Fig. 1.27. Two angles of projection for the same range

1.20. TWO ANGLES OF PROJECTION FOR THE SAME RANGE

$$\text{Again, } R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin (180° - 2\theta)}{g} \qquad [\because \sin (180° - 2\theta) = \sin 2\theta]$$

or $$R = \frac{u^2 \sin 2(\theta)}{g}$$

$$= \frac{u^2 \sin 2(90° - \theta)}{g}$$

This shows that there are two angles of projection for the same horizontal range *i.e.,* θ and (90° – θ) with the horizontal. The projectile will cover the same horizontal range whether it is thrown at an angle θ or (90° – θ) with the horizontal.

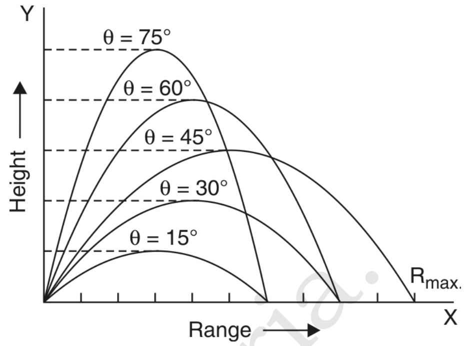


Fig. 1.28. R is same for θ = 15° and 75°. Again, R is same for θ = 30° and 60°. R is maximum for θ = 45°

Example 2. *A projectile is thrown at an angle θ with the horizontal with kinetic energy E. Calculate the potential energy at the topmost point of the trajectory.*

Solution. Potential energy at the topmost point of the trajectory

$$= mg\,h_{max.} = mg\,\frac{u^2 \sin^2\theta}{2g}$$

$$= \left(\frac{1}{2}mu^2\right)\sin^2\theta = \mathbf{E \sin^2\theta}$$

Example 3. *A projectile is thrown with an initial velocity of* $x\hat{i} + y\hat{j}$. *The range of the projectile is twice the maximum height of the projectile. Calculate* $\frac{y}{x}$.

Solution. $\frac{u^2 \sin 2\theta}{g} = 2\,\frac{u^2 \sin^2\theta}{2g}$

or $2u^2 \sin\theta\cos\theta = u^2\sin^2\theta$

or $2(u\sin\theta)(u\cos\theta) = (u\sin\theta)(u\sin\theta)$

But $u\sin\theta = y$ and $u\cos\theta = x$ (given)

$\therefore$ $2yx = y^2$ or $2x = y$ or $\frac{y}{x} = \mathbf{2}$

1.21. UNIFORM CIRCULAR MOTION

When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion. The word "uniform" refers to the speed which is uniform (constant) throughout the motion.

Consider an object moving with uniform speed v in a circle of radius R as shown in Fig. 1.29. Since the velocity of the object is changing continuously in direction, therefore, the object undergoes acceleration. Let us find the magnitude and direction of this acceleration.

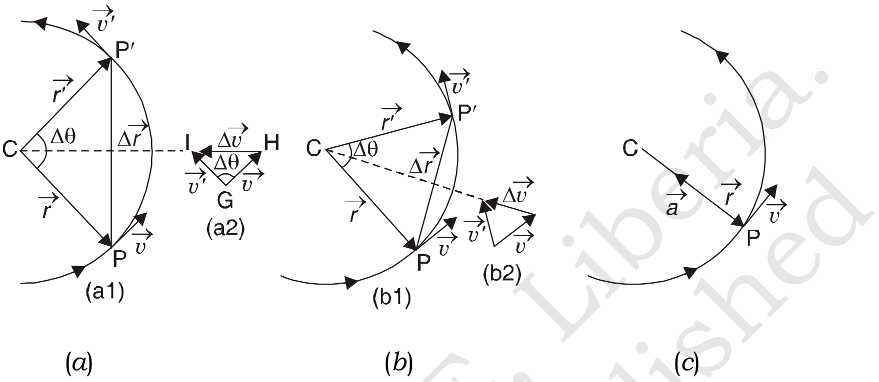


Fig. 1.29. Velocity and acceleration of an object in uniform circular motion. The time interval Δt decreases from (*a*) to (*c*) where it is zero. The acceleration is directed, at each point of the path, towards the centre of the circle

Let $\vec{r}$ and $\vec{r}'$ be the position vectors and $\vec{v}$ and $\vec{v}'$ the velocities of the object when it is at point P and P′ as shown in Fig. 1.29(*a*). By definition, velocity at a point is along the tangent at that point in the direction of motion. The velocity vectors $\vec{v}$ and $\vec{v}'$ are as shown in Fig. 1.29(*a*). $\Delta\vec{v}$ is obtained in Fig. 1.29(*b*) using the triangle law of vector addition. Since the path is circular, $\vec{v}$ is perpendicular to $\vec{r}$ and so is $\vec{v}'$ to $\vec{r}'$. Therefore, $\Delta\vec{v}$ is perpendicular to $\Delta\vec{r}$. Since average acceleration is along $\Delta\vec{v}$ $\left(\vec{a}_{av.} = \dfrac{\Delta\vec{v}}{\Delta t}\right)$, the average acceleration $\vec{a}_{av.}$ is perpendicular to $\Delta\vec{r}$. If we place $\Delta\vec{v}$ on the line that bisects the angle between $\vec{r}$ and $\vec{r}'$, we see that it is directed towards the centre of the circle. Figure 1.29(*b*) shows the same quantities for smaller time interval. $\Delta\vec{v}$ and hence $\vec{a}_{av.}$ is again directed towards the centre. In Fig. 1.29(*c*), $\Delta t \to 0$ and the average acceleration becomes the instantaneous acceleration. It is directed towards the centre.

The magnitude of $\vec{a}$ is, by definition, given by

$$|\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{v}|}{\Delta t}$$

Let the angle between position vectors $\vec{r}$ and $\vec{r'}$ be $\Delta\theta$. Since the velocity vectors $\vec{v}$ and $\vec{v'}$ are always perpendicular to the position vectors, the angle between them is also $\Delta\theta$. Therefore, the triangle CPP′ formed by the position vectors and the triangle GHI formed by the velocity vectors $\vec{v}$, $\vec{v'}$ and $\Delta\vec{v}$ are similar [Fig. 1.29(*a*)]. Therefore, the ratio of the base-length to side-length for one of the triangles is equal to that of the other triangle. That is:

$$\frac{|\Delta \vec{v}|}{v} = \frac{|\Delta \vec{r}|}{\mathrm{R}} \quad \text{or} \quad |\Delta \vec{v}| = v\frac{|\Delta \vec{r}|}{\mathrm{R}}$$

Therefore, $$|\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v\,|\Delta \vec{r}|}{\mathrm{R}\,\Delta t} = \frac{v}{\mathrm{R}} \lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t}$$

If Δt is small, $\Delta\theta$ will also be small and then arc PP′ can be approximately taken to be $|\Delta \vec{r}|$.

$$|\Delta \vec{r}| \cong v\Delta t$$

$$\frac{|\Delta \vec{r}|}{\Delta t} \cong v$$

or $$\lim_{\Delta t \to 0} \frac{|\Delta \vec{r}|}{\Delta t} = v$$

Therefore, the centripetal acceleration a_c is:

$$a_c = \left(\frac{v}{\mathrm{R}}\right) v = \frac{v^2}{\mathrm{R}}$$

Thus, the acceleration of an object moving with speed v in a circle of radius R has a magnitude $\frac{v^2}{\mathrm{R}}$ and is always **directed towards the centre.** This is why this acceleration is called **centripetal acceleration** (a term proposed by Newton). Since v and R are constant, the magnitude of the centripetal acceleration is also constant. However, the direction changes—pointing always towards the centre. Therefore, a centripetal acceleration is not a constant vector.

As the object moves from P to P′ in time Δt (= $t' - t$), the line CP (Fig. 1.29.) turns through an angle $\Delta\theta$ as shown in the figure. $\Delta\theta$ is called angular distance. We define the angular speed ω (Greek letter omega) as the time rate of change of angular displacement.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Now, if the distance travelled by the object during the time Δt is Δs, *i.e.,* PP′ is Δs, then:

$$v = \frac{\Delta s}{\Delta t}$$

but $\Delta s = R\,\Delta\theta$. Therefore:

$$v = R\frac{\Delta\theta}{\Delta t} = R\omega$$

We can express centripetal acceleration a_c in terms of angular speed:

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

$$a_c = \omega^2 R$$

The time taken by an object to make one revolution is known as its time period T and the number of revolutions made in one second is called its frequency $\nu\left(=\frac{1}{T}\right)$. However, during this time the distance moved by the object is, $s = 2\pi R$.

Therefore, $v = \dfrac{2\pi R}{T} = 2\pi R\nu$

In terms of frequency ν, we have

$$\omega = 2\pi\nu$$

$$v = 2\pi R\nu$$

$$a_c = 4\pi^2\,\nu^2 R.$$

Example 4. *A constant torque is acting on a wheel. If starting from rest, the wheel makes n rotations in t second, show that the angular acceleration is given by* $\alpha = \dfrac{4\pi n}{t^2}$ *rad* s^{-2}.

Solution. Since the wheel starts from rest, therefore, the initial angular velocity ω_0 is zero.

Number of rotations in t second = n

Angular displacement in time t, $\theta = 2\pi n$

Now, $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

or $$2\pi n = 0 + \frac{1}{2}\alpha t^2 \quad \text{or} \quad \alpha = \frac{4\pi n}{t^2} \ \textbf{rad s}^{-2}$$

1.22. OSCILLATORY MOTION

*If a body moves back and forth repeatedly about a mean position, it is said to possess **oscillatory** or **vibratory motion.***

Very often the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position, no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from this position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to **oscillations** or **vibrations**. For example, a ball placed in a bowl will be in equilibrium at the bottom. If displaced a little from the point, it will perform oscillations in the bowl.

Examples. (*i*) Motion of the pendulum of a wall clock. **(*ii*)** Vibrations of the wire of a '*sitar*'. **(*iii*)** Vibrations of the drum of a '*tabla*'. **(*iv*)** Oscillations of a mass suspended from a spring. **(*v*)** Motion of liquid in a U-tube when the liquid is once compressed in one limb and then left to itself. **(*vi*)** A weighted test tube floating in a liquid executes oscillatory motion when pressed down and released.

Difference between periodic motion and oscillatory motion. An oscillatory motion is always periodic. A periodic motion may or may not be oscillatory. So, oscillatory motion is merely a special case of periodic motion. As an example, the motion of the planets around the Sun is periodic but not **oscillatory.**

Difference between oscillations and vibrations. There is no significant difference between oscillations and vibrations. When the frequency is small, we use the term "oscillation" (like the oscillation of a branch of a tree). When the frequency is high, we use the term "vibration" (like the vibration of a string of a musical instrument).

Simplest form of oscillatory motion. The simplest form of oscillatory motion is *simple harmonic motion.* This motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position (equilibrium position). At any point in its oscillation, this force is directed towards the mean position.

1.23. SIMPLE HARMONIC MOTION

Simple Harmonic Motion is a motion which is necessarily periodic and oscillatory about a fixed mean position. A particle executing such a motion is always in *stable equilibrium* about its mean position. So, if a particle is disturbed slightly from its mean position, it tends to return to its mean position. The force which tends to bring the particle back to the mean position is called the *restoring force.* The greater the displacement of the particle from the mean position, greater is the restoring force. Thus, ***simple harmonic motion*** *is defined as such an oscillatory motion about a fixed point (mean position) in which the restoring force is always proportional to the displacement from that point and is always directed towards that point.*

If a particle suffers a small displacement x from its mean position, then the magnitude of restoring force F is given by

$$F = -kx \qquad \ldots(1)$$

where k is known as the *force constant.* Its SI unit is N m^{-1}. Its dimensional formula is $[ML^{\circ} T^{-2}]$. The negative sign in equation (1) indicates that the restoring force is directed towards the mean position.

Importance of the study of simple harmonic motion. Any periodic motion can be expressed as the resultant of two or more simple harmonic motions. So, simple harmonic motion is the simplest and most fundamental of all types of periodic motions.

1. In mechanical wave motion, the particles of the medium execute either simple harmonic motion or a combination of simple harmonic motions.
2. The vibrations of the air columns and strings of musical instruments are either simple harmonic or a superposition of simple harmonic motions.

3. The prongs of a vibrating tuning fork oscillate simple harmonically.

1.24. ROTATIONAL MOTION

Consider a rigid body which is so constrained that it cannot have translational motion. The only possible motion of such a rigid body is **rotation.** The line along which the body is fixed is termed as its **axis of rotation.** If you look around, you will come across many examples of rotation about an axis, a ceiling fan, a potter's wheel, a giant wheel in a fair, a merry-go-round and so on [Fig. 1.30(*a*) and (*b*)].

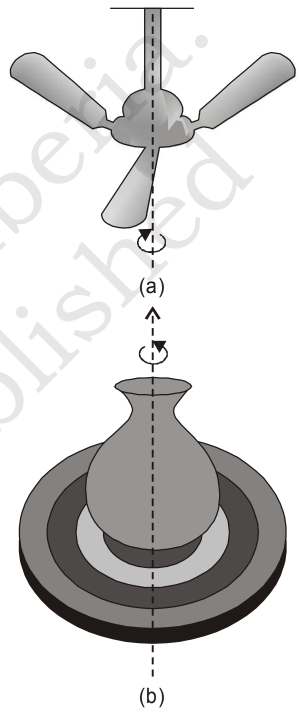


Fig. 1.30. Rotation about a fixed axis (*a*) A ceiling fan (*b*) A potter's wheel

In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis. Fig. 1.31 shows the rotational motion of a rigid body about a fixed axis (the z-axis of the frame of reference). Let P_1 be a particle of the rigid body, arbitrarily chosen and at a distance r_1 from fixed axis. The particle P_1 describes a circle of radius r_1 with its centre C_1 on the fixed axis. The circle lies in a plane perpendicular to the axis. The figure also shows another

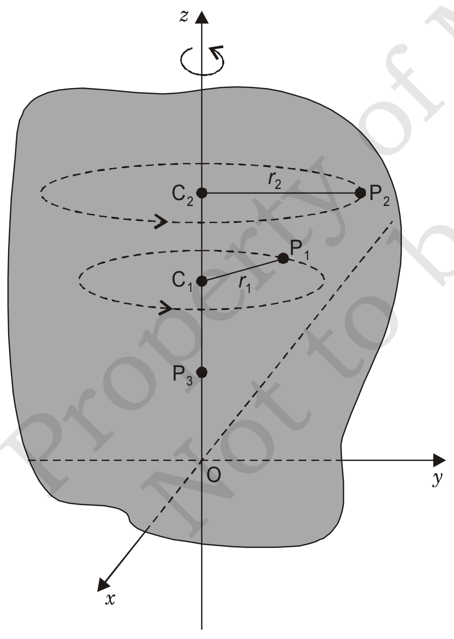


Fig. 1.31. A rigid body rotation about the z-axis (Each point of the body such as P_1 or P_2 describes a circle with its centre (C_1 or C_2) on the axis. The radius of the circle (r_1 or r_2) is the perpendicular distance of the point (P_1 or P_2) from the axis. A point on the axis like P_3 remains stationary.)

particle P_2 of the rigid body, P_2 is at a distance r_2 from the fixed axis. The particle P_2 moves in a circle of radius r_2 and with centre C_2 on the axis. This circle, too, lies in a plane perpendicular to the axis. Note that the circles described by P_1 and P_2 may lie in different planes; both these planes, however, are perpendicular to the fixed axis. For any particle on the axis like P_3, $r = 0$. Any such particle remains stationary while the body rotates. This is expected since the axis is fixed.

In some examples of rotation, however, the axis may not be fixed. A prominent example of this kind of rotation is a top spinning in place [Fig. 1.32.]. (We assume that the top does not slip from place to place and so does not have translational motion.) We know from experience that the axis of such a spinning top moves around the vertical through its point of contact with the ground, sweeping out a cone as shown in Fig. 1.32. (This movement of the axis of the top around the vertical is termed **precession.)** Note, the **point of contact of the top with ground is fixed.** The axis of rotation of the top at any instant passes through the point of contact. Another simple example of this kind of rotation is the oscillating table fan or a pedestal fan. You may have observed that the axis of rotation of such a fan has an oscillating (sidewise) movement in a horizontal plane about the vertical through the point at which the axis is pivoted [point O in Fig. 1.33].

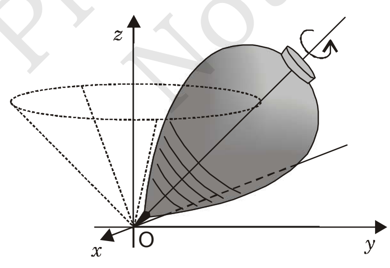


Fig. 1.32. (*a*) A spinning top (The point of contact of the top with the ground, its tip O, is fixed)

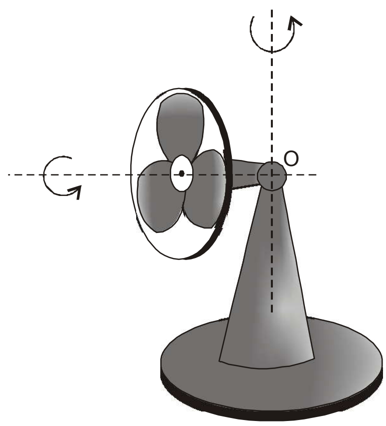


Fig. 1.33. (*b*) An oscillating table fan. The pivot of the fan, point O, is fixed

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A body moves in a plane so that the displacements along the x and y axes are given by $x = 3t^3$ and $y = 4t^3$. The velocity of the body is

(*a*) $9\ t$ (*b*) $15\ t$

(*c*) $15\ t^2$ (*d*) $25\ t^2$.

2. A particle is travelling along a straight line OX. The distance x (in metre) of the particle from O at a time t is given by $x = 37 + 27t - t^3$ where t is time in second. The distance of the particle from O when it comes to rest is

(*a*) 81 m (*b*) 91 m

(*c*) 101 m (*d*) 111 m.

3. A bullet on penetrating 30 cm into its target loses its velocity by 50%. What additional distance will it penetrate into the target before it comes to rest?

(*a*) 30 cm (*b*) 20 cm

(*c*) 10 cm (*d*) 5 cm.

4. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m s^{-1} where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m s^{-2}, the equation of its trajectory is

(*a*) $4y = 2x - 25x^2$ (*b*) $y = x - 5x^2$

(*c*) $y = 2x - 5x^2$ (*d*) $4y = 2x - 5x^2$

5. The distance x covered by a particle varies with time t as $x^2 = 2t^2 + 6t + 1$. Its acceleration varies with x as

(*a*) x (*b*) x^2

(*c*) x^{-1} (*d*) x^{-3}

B. Fill in the Blanks

1. In the entire path of a projectile, the quantity that remains unchanged is __________ .

2. Among the following, the vector quantity is __________ .

3. If the velocity (in m s^{-1}) of a particle is given by $4.0\hat{i} + 5.0t\hat{j}$, then the magnitude of its acceleration (in m s^{-2}) is __________ .

4. The horizontal range of a projectile is maximum when the angle of projection is __________ .

5. The graph between displacement and time for a particle moving with uniform acceleration is a __________ .

C. Very Short Answer Questions

1. Name a quantity which remains unchanged during the flight of an oblique projectile.
2. At which point of the projectile path, the speed is minimum?
3. Name five physical quantities which change during the motion of an oblique projectile.
4. A body is projected so that it has maximum range R. What is the maximum height reached during the flight?
5. Name two quantities which would be reduced if air resistance is taken into account in the study of motion of oblique projectile.

D. Short Answer Questions

1. A ball is thrown horizontally and at the same time another ball is dropped from the top of a tower. (*i*) Will both the balls hit the ground with the same velocity? (*ii*) Will both the balls reach the ground at the same time?
2. What is the effect of air resistance on the time of flight and horizontal range of the projectile?
3. A projectile of mass m is projected with velocity v at an angle θ with the horizontal. What is the magnitude of the change in momentum of the projectile after time t?
4. The maximum horizontal range of a cannon is 4 km. What is the muzzle velocity of the shell, if g = 10 m s^{-2} ?
5. Why does a tennis ball bounce higher on hills than in plains?

E. Long Answer Questions

1. A motorboat is racing towards north at 25 km h^{-1} and the water current in that region is 10 km h^{-1} in the direction of 60° east of south. Find the resultant velocity of the boat.

2. Two vectors acting in opposite directions have a resultant of 10 units. If they act at right angles to each other, the resultant is 50 units. Calculate the magnitudes of the two vectors.

3. A car travelling at 20 m s^{-1} due north along the highway makes a right turn on to a side road that heads due east. It takes 50 s for the car to complete the turn. At the end of 50 second, the car has a speed of 15 m s^{-1} along the side road. Determine the magnitude of average acceleration over the 50 second interval.

4. A child pulls a rope attached to a stone with a force of 60 N. The rope makes an angle of 40° to the ground.
   (a) Calculate the effective value of the pull tending to move the stone along the ground.
   (b) Calculate the force tending to lift the stone vertically.

5. Referred to two rectangular axes, the three successive displacement vectors have components of + 2.4 m, + 0.5 m ; – 4.6 m, + 3.3 m ; and – 2.8 m, – 15.8 m. Calculate the components of the resultant displacement. What is the magnitude of the resultant?